**Methods**

The purpose of this “toy example” is to illustrate the potential for approximating equilibrium quantities within a spatially-structured assessment model. [Detail here on how analytical solution impossible].

*Basic Structure*

To illustrate the various approaches to calculating spatial reference points described above, we constructed a simple age-structured model with three spatial areas where a survival equation links age classes and gear-selectivity is specified for three fleets, each targeting a single area (similar to Sampson, 2014). The basic model does not consider growth. Ages are modelled from 0 (recruits) to 20+. Mortality is fixed for all regions at 0.15. Selectivity-at-age a is dome-shaped, and fleet specific (Figure 1).  Movement is governed by a probability transition matrix between areas at each age, . The matrix specifies unidirectional movement from areas 1 and 2 to area 3 at varying proportions for fish below age 6, after which fish remain in their respective area (Figure 2).The numbers-at-age in each area are generated by the following:

Where is a vector of length i (number of areas) indicating the distribution of age-0 recruits to

each area; and

is the area- and age-specific fleet selectivity; and

F is the fishing mortality rate; and

is a user-defined reference recruitment level

Spawning biomass given F is obtained by multiplying the numbers-at-age by an expected weight at age and fecundity at age. This can be left as an area-specific value or summed to become total spawning biomass,

*Recruitment*

Equilibrium calculations assume a Beverton-Holt stock recruitment relationship with a shared value for steepness (h). The general form of this relationship is:

(2a)

(2b)

Where is the equilibrium spawning biomass;

SB0 is the unfished spawning biomass obtained by setting F to 0 in Equation 1;

*h* or steepness (the expected proportion of unfished recruitment at 20% unfished

spawning biomass) is 0.5; and

is unfished recruitment, set to 500.

*Equilibrium Calculations: Two Approaches*

Here we present two alternative methods for calculating equilibrium recruitment, biomass and yield under a given value of *F* (which is applied to all three areas). The first approach, referred to here as “current”, reflects the extant method used in Stock Synthesis when the user has specified a multi-area model. [More detail on current approach here]. The second approach, labeled “proposed”, aims to approximate the equilibrium conditions of a spatial model.

The essential difference between the two approaches is that the “proposed” approach considers the conditions within individual areas, and re-weights the distribution of age-0 recruits based on the ratio of equilibrium biomasses among areas. In practice, the proposed approach proceeds as follows:

1. Distribute to areas according to a nominal recruitment distribution,
2. Iterating among areas within ages, calculate the survivors-at-age, and mix them according to the movement matrix (Equation 1). Until this point, both approaches are identical.
3. Note both the area-specific and total spawning biomass and yield.
4. Calculate spawner-biomass-per-recruit and yield-per-recruit for each area, using and , i.e.

;

1. Use Equation 2 to calculate the expected recruitment from each area, using the area-specific unfished biomass and ; unfished recruitment is then
2. Once equilibrium values for all areas have been found, adjust by the ratio
3. Repeat steps 1-5, using in lieu of .

*Results*

TBD

**Tables**

|  |  |  |  |
| --- | --- | --- | --- |
| **Symbol** | **Description** | **Current Approach** | **Proposed Approach** |
|  | Distribution of age-0 recruits among areas | Area 1 = 0.5  Area 2 = 0.3  Area 3 = 0.2 | Adjusted based on ratio of B equilibrium |
| h | expected proportion of unfished recruitment at 20% unfished spawning biomass | 0.5 | unchanged |
|  | Unfished recruitment | 500 (global) | unchanged |
|  | Reference recruitment | 100 (global) | unchanged |

Table 1. Input parameter values for both the current and proposed approaches.

**Figures**

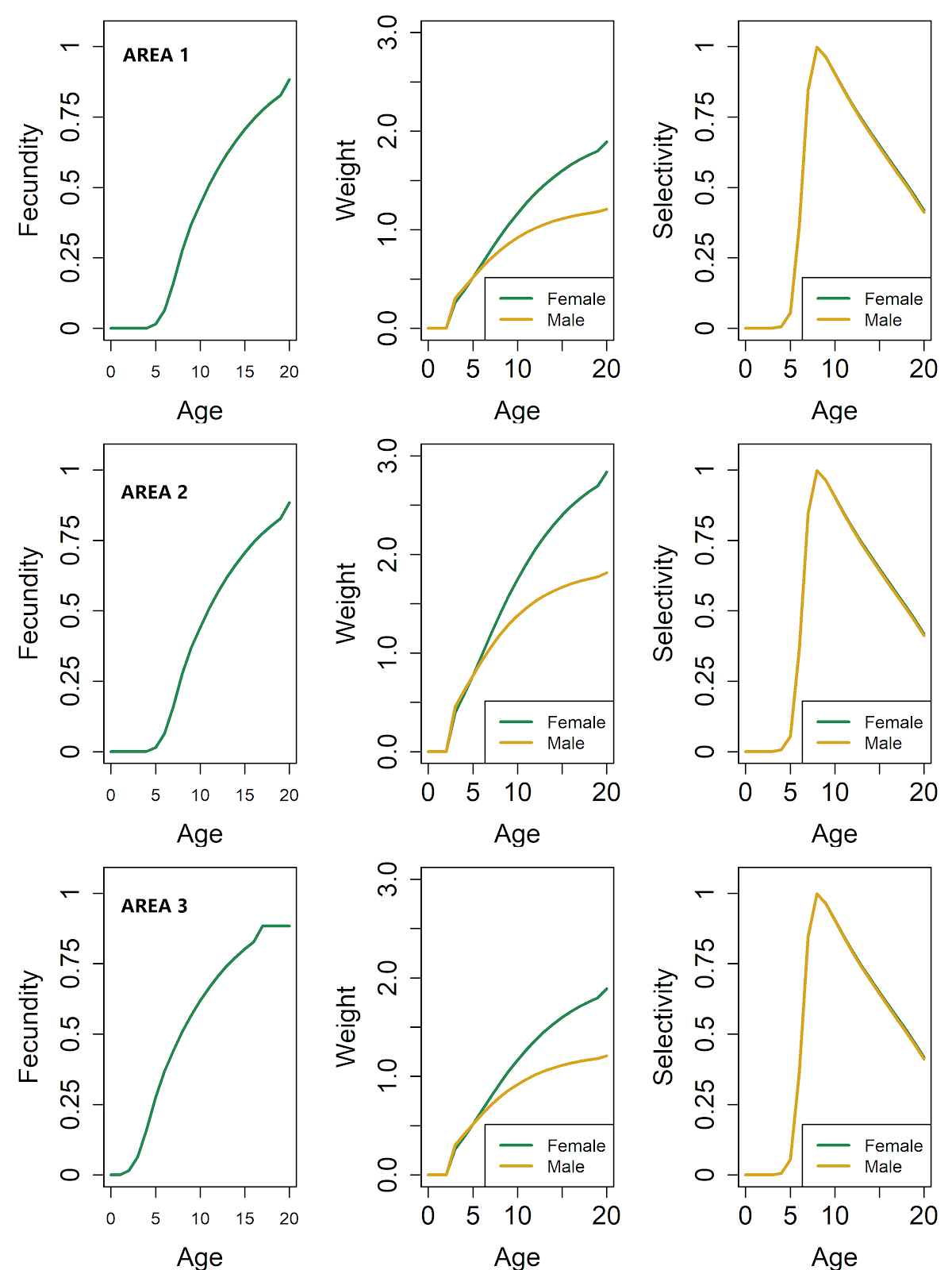


Figure 1. Input female fecundity-at-age (leftmost column), weight-at-age (central column), and fishery gear selectivity (rightmost column) for females (green lines) and males (gold lines) in each of three spatial areas (rows).

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Description automatically generated

Figure 2. Input probability transition matrices among areas by age. Movement is unidirectional from areas 1 and 2 to area 3 for fish under age 6 and ceases thereafter. Lighter colors indicate higher movement probabilities; value within cell indicates the probability of moving from the source area to the sink area.

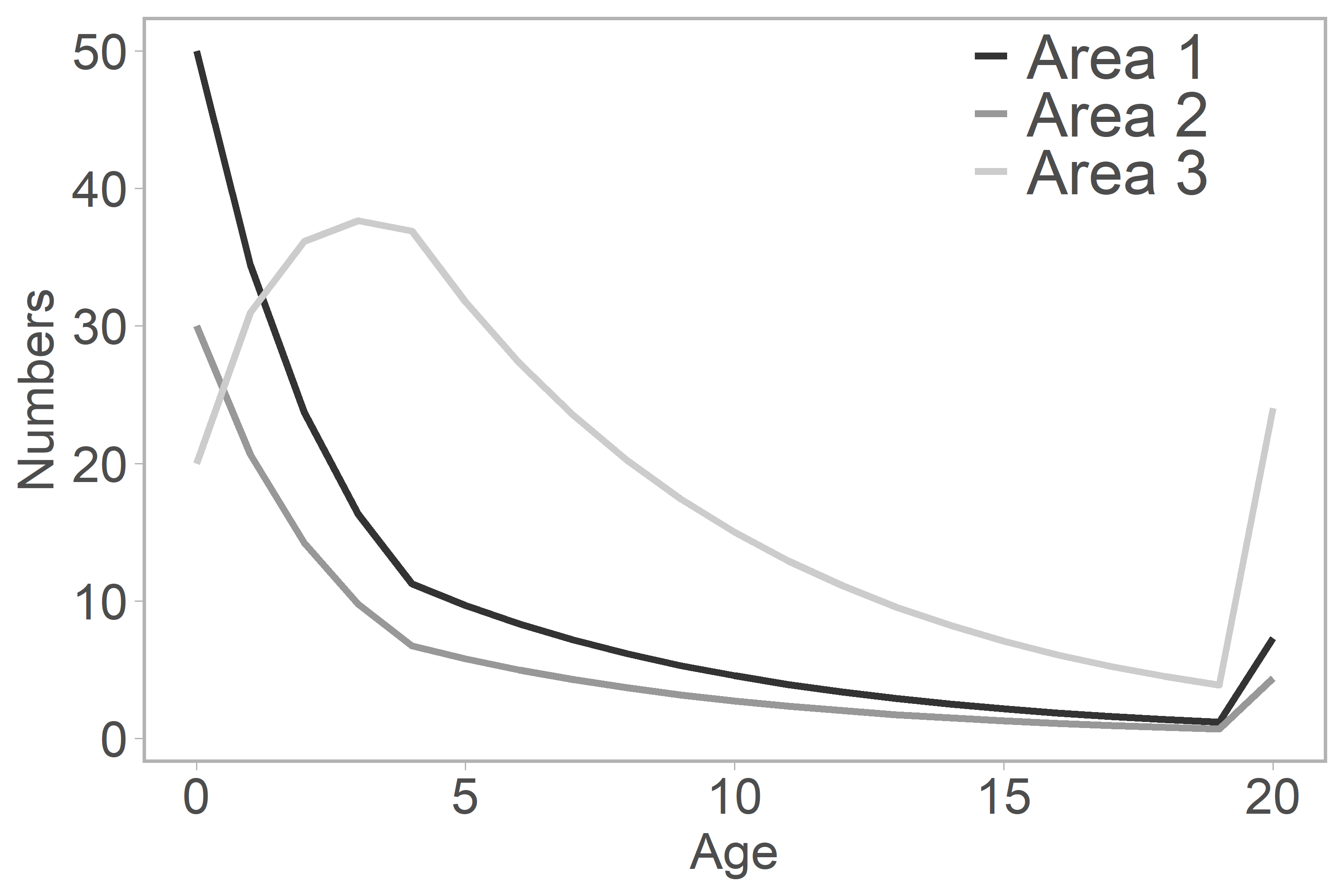


Figure 3. Unfished numbers-at-age among areas. The unidirectional movement dynamic acts as a subsidy of individuals to Area 3.

Figure 4. Comparison of “current” and “proposed” approaches for calculating